JP Morgan Startup Survival Rates
The Discrete-Time Survival Rate Curve

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In this white paper we will calculate the conditional probability of company failure over the time interval $[s,t]$ given that the company has survived over the time interval $[0,s]$. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

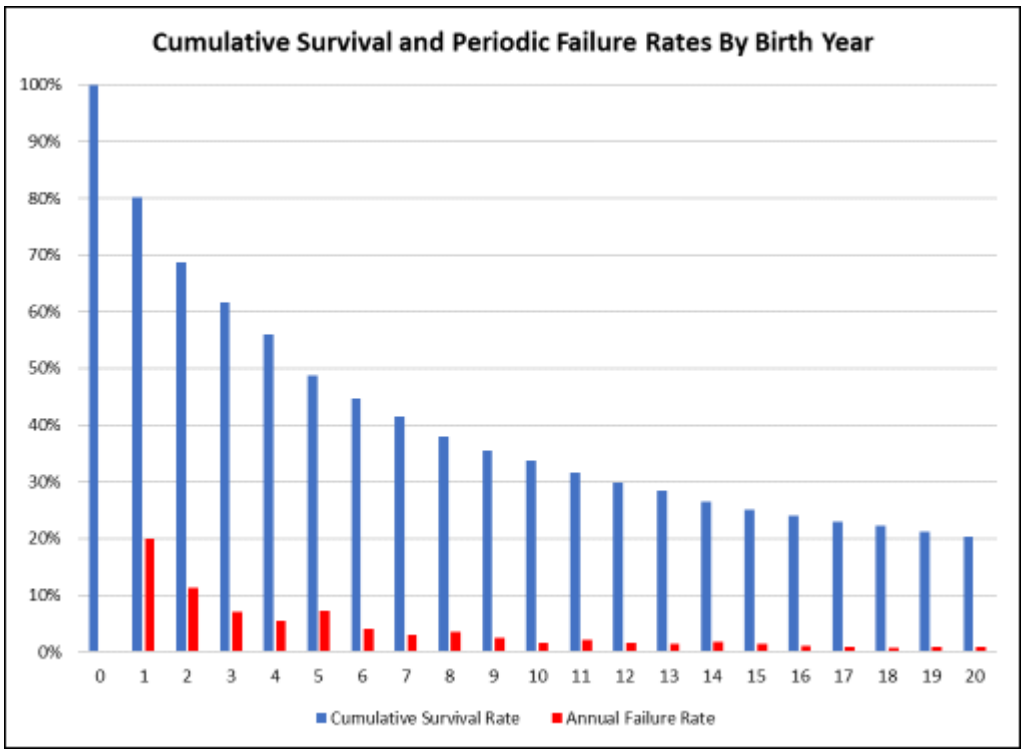
ABC Company is a technology startup that is now 3.25 years old. We are tasked with structuring a 4.50 year loan guarantee and want to know the probability of ABC Company's failure over the guarantee term. We are given the following startup survival and failure rates by birth year...

Source: https://www.jpmorganchase.com/institute/research/small-business/small-business-dashboard/longevity

Table 1: JP Morgan Data

Birth Year	Survival Rate	Failure Rate
0	1.0000	–
1	0.8010	0.1990
2	0.6870	0.1140
3	0.6160	0.0710
4	0.5600	0.0560
5	0.4880	0.0720
6	0.4470	0.0410
7	0.4160	0.0310
8	0.3800	0.0360
9	0.3550	0.0250
10	0.3380	0.0170
11	0.3160	0.0220
12	0.2990	0.0170
13	0.2840	0.0150
14	0.2660	0.0180
15	0.2510	0.0150
16	0.2400	0.0110
17	0.2300	0.0100
18	0.2220	0.0080
19	0.2130	0.0090
20	0.2040	0.0090
Total	–	0.7960

Table 2: Survival And Failure Rate Graph By Birth Year



Question: Using the data above, what is ABC Company's probability of failure over the guarantee term?

Conditional Probability of Failure

We will define the function $S(t)$ to be the survival function at time t, which is the probability that a startup company survives over the time interval $[0, t]$ where time zero is the time in years that the company began operations. Noting that the data in Table 1 above is in annual increments, the equation for the interpolated survival rate at time t where T_B is the annual rate at the beginning of the time interval and T_E is the annual rate at the end of the time interval is...

$$S(t) = S(T_B) + \frac{S(T_E) - S(T_B)}{T_E - T_B} \times (t - T_B) \;\; \text{...where...} \;\; T_B \leq t \leq T_E \tag{1}$$

Using the data in Table 1 above the survival function for year 3 and year 4 are...

$$\text{Survival function}: \;\; S(3.00) = 0.6160 \;\; \text{...and...} \;\; S(4.00) = 0.5600 \tag{2}$$

If we are currently standing at time zero then using Equations (1) and (2) above the interpolated probability that our company (ABC Company) survives over the time interval $[0, 3.25]$ is...

$$S(3.25) = S(3.00) + \frac{S(4.00) - S(3.00)}{4.00 - 3.00} \times (3.25 - 3.00) = 0.6160 + \frac{0.5600 - 0.6160}{1.00} \times 0.25 = 0.6020 \tag{3}$$

Using the data in Table 1 above the survival function for year 7 and year 8 are...

$$\text{Survival function}: \;\; S(7.00) = 0.4160 \;\; \text{...and...} \;\; S(8.00) = 0.3800 \tag{4}$$

If we are currently standing at time zero then using Equations (1) and (4) above the interpolated probability that our company (ABC Company) survives over the time interval $[0, 3.25 + 4.50\,\text{guarantee term}]$ is...

$$S(7.75) = S(7.00) + \frac{S(8.00) - S(7.00)}{8.00 - 7.00} \times (7.75 - 7.00) = 0.4160 + \frac{0.3800 - 0.4160}{1.00} \times 0.75 = 0.3890 \tag{5}$$

We will define the function $P[t]$ to be the probability that the company survives over the time interval $[0, t]$. We will define the function $P[t|s]$ to be the **conditional probability** that the company survives over the time interval $[s, t]$ **given that** the company has survived (past tense) over the time interval $[0, s]$. The conditional probability equation below applies to our company...

$$P\Big[7.75\Big] = P\Big[3.25\Big] \times P\Big[7.75\,\Big|\,3.25\Big] \;\; \text{...such that...} \;\; P\Big[7.75\,\Big|\,3.25\Big] = P\Big[7.75\Big] \div P\Big[3.25\Big] = \frac{0.3890}{0.6020} = 0.6462 \tag{6}$$

The answer to Equation (6) above is the conditional probablity of company survival over the guarantee term given that the company survived up to the beginning of the guarantee term. The conditional probability of failure over the guarantee term (the answer to our hypothetical problem) is therefore...

$$\text{Prob}\Big[\text{Failure over time interval } [3.25, 7.75]\,\Big|\,\text{Survival over time interval } [0, 3.25]\Big] = 1 - 0.6462 = 0.3538 \tag{7}$$